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ELECTRICAL MEASUREMENTS

PART II

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ELECTRICAL MEASUREMENTS

PART II—ADVANCED

INSTRUCTION PAPER

PREPARED BY

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ELECTRICAL MEASUREMENTS

PART II—ADVANCED

MEASUREMENT OF CAPACITY

Ballistic Galvanometer. In the measurement of the capacity of a condenser by the methods given in the subsequent pages, the charge of electricity from the condenser is allowed to flow as a momentary current through a galvanometer, giving the suspension a sudden *kick*. In order to calculate from this deflection the *quantity of electricity* in the condenser, it is necessary to assume that the galvanometer suspension is so heavy that it will not have moved very far before the charge has completely passed. This requisite, viz, a heavy suspension, is the distinguishing feature of the *ballistic* type of galvanometer. (See Fig. 7, Part I.)

As a rule the methods of measurement involve only a comparison of the deflections produced in the ballistic galvanometer by charged condensers of known and unknown capacity, so that, as long as the capacity of a standard condenser is known, the unknown factors, the galvanometer constant, etc., are unimportant. Nevertheless it may be instructive to know how these unknowns can be determined and the deflections can be made to give the actual quantity of electricity in the given condensers.

Because of the fact that the deflection of the galvanometer is not proportional to the current which produces the deflection, it is necessary to know the factor called the *constant* of the galvanometer before measurements can be taken. This constant is used in various forms but can be briefly stated as *the constant ratio between the current and the deflection produced by it.* When put in more definite form it can be given as follows:

$$K = \frac{2\,I\,D}{d}$$

in which I is the current flowing in the galvanometer, D is the distance

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from the galvanometer mirror to the scale, and d is the deflection produced on the scale.

With this in mind let us consider *how to find the quantity of electricity Q from the throw θ of the galvanometer, the galvanometer contant K, and the half period of the suspension.*

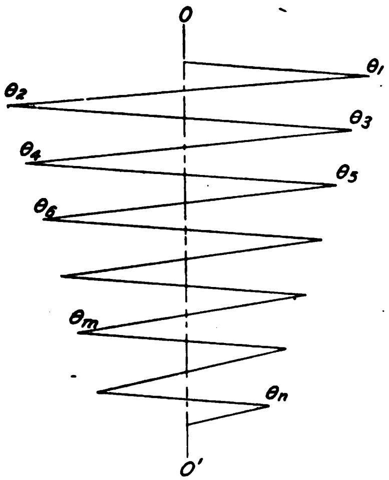

Fig. 36. Damping Diagram.

As has been stated above, while the quantity Q is passing through the galvanometer, short though it may be in duration, it constitutes a current and the magnetic effect of this current exerts a turning moment on the coil.

If I represents the mean value of this current, then the mean moment of force $\overline{Fh}$ acting on the coil while the current is flowing is

$$\overline{Fh} = I\,\mathrm{H}\,A$$

in which H is the strength of the field and A is the area of the galvanometer coil. If τ is the duration of the discharge, then the moment of force times the time can be given by

$$\overline{Fh}\tau = I\tau\mathrm{H}A = Q\mathrm{H}A$$

in which Q is the quantity of electricity, equal to $I\tau$.

If the moment of inertia M of the suspension and the angular velocity, which is given to it by this *kick,* are taken into consideration, the quantity of electricity Q may be obtained from the above equation as follows:

$$Q = K\omega\frac{M}{T_0}$$

in which ω is the angular velocity and T_0 is the torsion constant of the suspension. By taking the half period of the suspension, which is easily obtained by counting the time of a given number of swings, and expressing ω in terms of the angle of throw θ, the expression for the quantity of electricity is given by the following equation:

$$Q = \frac{K\theta t}{\pi}$$

in which K is the galvanometer constant, θ the angle of throw (obtained by dividing the deflection d by twice the distance from mirror to scale D), t the half period of the suspension, and π 3.1416.

For accurate work θ must be multiplied by a damping factor $\sqrt{\rho}$, derived as follows: With the suspension swinging freely, Fig. 36, take a deflection θ_m, then after a given number of swings (n-m) take another deflection, θ_n; ρ is the (n-m) root of the ratio $\frac{\theta_m}{\theta_n}$.

Condensers. A condenser consists in its simplest form of two metal sheets separated by a nonconducting material, Fig. 37. If an e. m .f. is applied to the two metal sheets, they will take a static charge, one positive and the other negative. The nonconducting material is called a *dielectric*, as the electric force acts through it (*dia* meaning through). The capacity of such a condenser is proportional to the area of the sheets and inversely proportional to the thickness of the dielectric. If the condenser is in the form of a glass jar coated outside and inside with tin foil, the arrangement is called a *Leyden jar*. A considerable portion of the surface near the edges of the jar should be free from the tin foil coating in order that the charge may not leak over the surface of the glass. The best condensers are made of many sheets of mica with sheets of tin foil interlarded, every alternate one being connected to one, and the others to the other terminal of the condenser, Fig. 38. By using many sheets of tin foil the capacity is increased in proportion to the total area. Mica is an excellent material for the dielectric as its resistance is extremely high, and very thin sheets have enough strength to withstand the mechanical stress

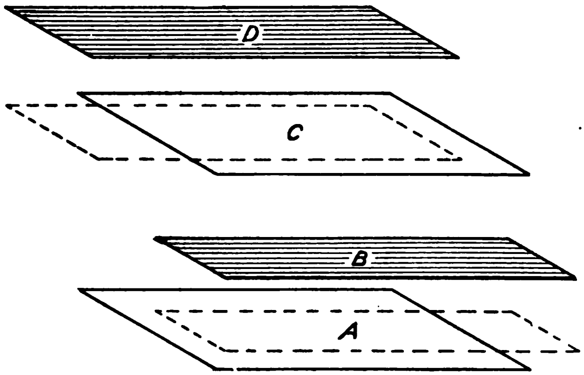

Fig. 37. Condenser Sheets.

due to the electric charges without breaking down. The mica and tin foil are clamped in place and the whole immersed in melted paraffin and then withdrawn, carrying out a coating of paraffin which protects the condenser from the effects of moisture. Several condensers of assorted capacities are frequently mounted in one box, Fig. 39. A 1 m. f. box will frequently have condensers of 0.5, 0.2, 0.1, 0.1, 0.05 and 0.05 microfarad, Fig. 40. Cheaper condensers have paraffined paper or other materials in place of mica; but are usually poor since the dielectric, though it does not break down, is apt to yield gradually to the strain of the charge, producing an effect which is known as *absorption of the charge.* It seems as though some of the charge had been lost for when the condenser is discharged, less charge comes out than was put in. It is true that real leakage causes a loss of part of the charge, but we find also that a poor condenser, if set aside after being discharged, will, on a later test, show a small charge which has come from the gradual return of the dielectric to its original state. The Leyden jar (glass dielectric) absorbs a considerable portion of its charge. Standard condensers are sometimes made with massive plates of metal and with air, which has no absorption, as the dielectric. They are very expensive and have small capacity. For practical purposes

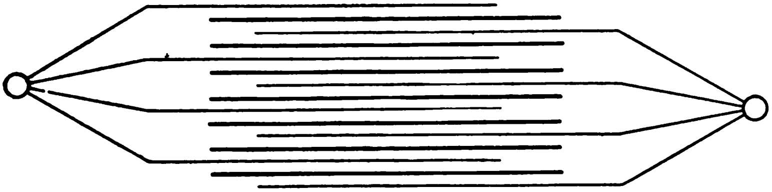

Fig. 38. Simple Condenser.

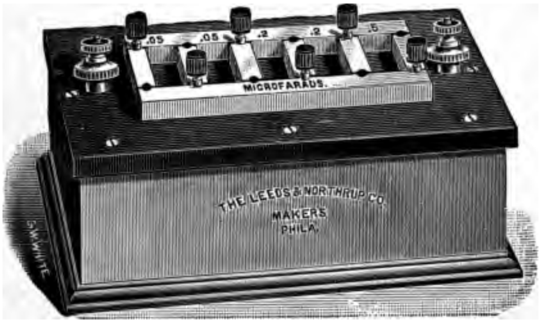

Fig. 39. Variable Condenser.

the best condensers have mica for the dielectric, for mica shows almost no absorption of the charge.

Single conductor submarine and land cables have the properties of condensers, the water acting as the second sheet in the case of the former, and the usual lead covering in the case of the latter. In telephone cables, each pair of wires and the insulation between make up a condenser. These cables almost always show considerable absorption of the charge.

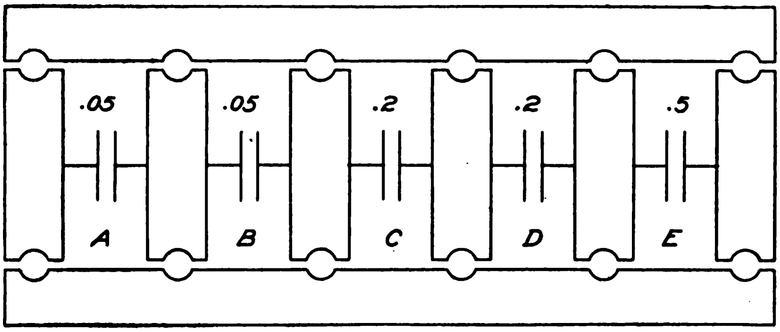

Fig. 40. Plan of Variable Condenser.

Direct Deflection Method. Two condensers may be compared as to their capacity, if first one and then the other is charged by a cell B of known e. m. f., and then discharged through a ballistic galvanometer G, Fig. 41. If the deflection with the standard of capacity C is D_1, and that with the unknown of capacity X is d_2 then

$$X = C\frac{d_2}{d_1}$$

A charge and discharge key K, Fig. 42, connects the condenser first to the battery and then to the galvanometer.

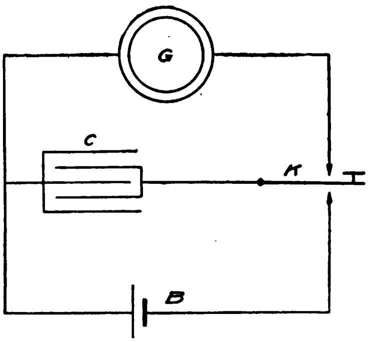

Fig. 41. Diagram for Direct Deflection Method.

This method is convenient, but the accuracy of the results depends on the accuracy with which the throw of the galvanometer can be read. The accuracy is not much better than 1% even if neither condenser shows absorption of charge. If either condenser absorbs charge, the ratio of the deflections will depend on the time of charge and the slowness of clearing out of the charge. There are better methods.

Example. With a condenser of 0.5 microfarads in circuit, the deflection is 46 divisions; with the unknown capacity X in circuit, the deflection is 69 divisions. What is X?

Ans. $X = 0.75$ m. f.

Bridge Methods. The Wheatstone's bridge method of comparing resistances may be adapted to the comparison of two capacities. The apparatus may be arranged as shown in either Fig. 43 or Fig. 44. In the former a charge and discharge key is used to charge and discharge the condensers. If all the charge taken by C_1 passes through R_1 and all taken by C_2 passes through R_2, both in charging and discharging, then the galvanometer will not deflect; otherwise it will deflect in opposite directions for the charge and discharge. As the current divides in parallel circuits in inverse proportion to the resistances; and as the charge, and therefore the current, taken by condensers in parallel is directly proportional to their capacities, it is evident that if no current passes through the galvanometer

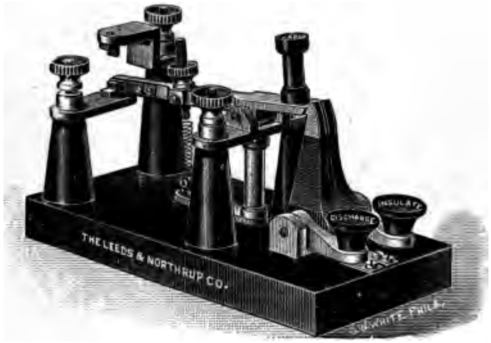

Fig. 42. Kempe Discharge Key.

$$C_1 : C_2 : : R_2 : R_1$$

therefore

$$C_1 = C_2 \frac{R_2}{R_1}$$

In Fig. 44 the battery circuit produces a current through R_1 and R_2, and they take potential differences between their terminals proportional to their resistances when the current becomes steady. The condensers will take a certain charge; and if the galvanometer is open, both condensers must take equal charges regardless of their capacities. If one has a smaller capacity than the other, the former will acquire a smaller potential difference than the latter, as the charge is equal to the product of the capacity and the potential difference. If C_1 and R_1 acquire equal potential differences, and C_2 and R_2 also equal potential differences, then on closing the galvanometer key no deflection will result. If on closing the galvanometer key a deflection results, it is evident that the above relation is not satisfied. If no deflection results

$$C_1 : C_2 : : R_2 : R_1,$$

and

$$C = C_2 \frac{R_2}{R_1}$$

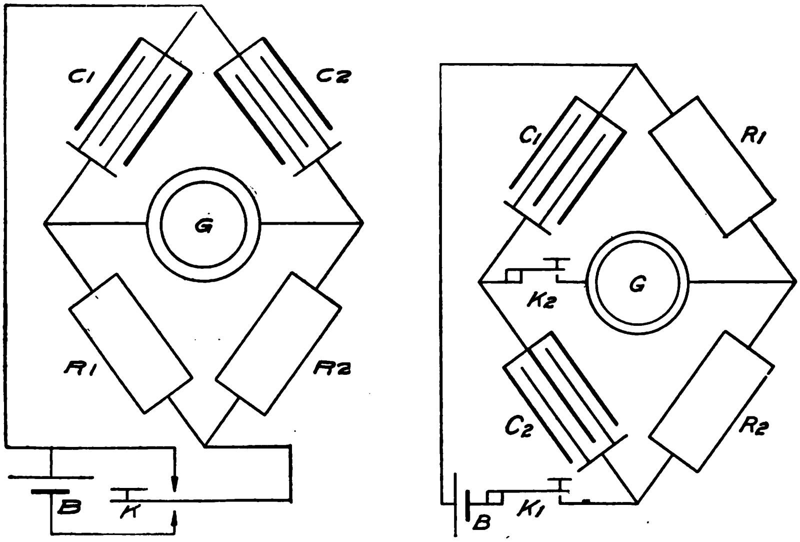

Fig. 43. Fig. 44.

Diagrams for Bridge Methods for Measuring Capacities of Condensers.

The order of closing and opening keys is important. It should be as follows: First close K_1; then K_2 and note deflection if any; second, open K_2, then K_1, then close K_2 and note deflection (or discharge) which should be in opposite direction. Then open K_2. It is necessary to discharge the condensers before recharging them, otherwise they will take no new charge beyond what is necessary to make up for leakage or absorption of charge. If the condensers absorb charges it is impossible to get a perfect balance,

Method of Mixtures. In the method of mixtures the positive charge taken by one condenser is mixed with the negative charge of the other, and *vice versâ,* and the remaining difference of charges is discharged through the galvanometer. This method allows the time of charge and the time of mixing the charges to be varied at will, thus allowing the absorbed charge more or less time to make itself felt.

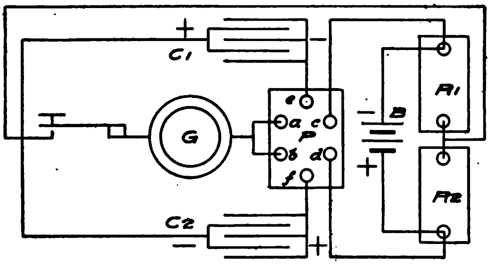

Fig. 45. Diagram for Method of Mixtures.

In Fig. 45, the Pohl's commutator *P*, by bringing the points *e* and *c* and *f* and *d* into contact as indicated, allows the condenser C_1 to charge until its potential difference is equal to that over R_1 (due to the current from the battery *B*), and also allows C_2 to charge until its potential difference equals that over R_2. The commutator is then reversed, bringing *e* into contact with *a*, and *f* into contact with *d*. The points *a* and *b* are permanently connected together. The + charge on C_2 can mix with the − charge on C_1, and the other charges on C_1 and C_2—which were previously so-called *bound charges*—now become free and can mix also. The remaining charges are divided between the two condensers in proportion to their capacities. If now the key *K* is closed, these remaining charges are discharged through the galvanometer. If no deflection occurs the charge remaining must be *nil.* As the charge taken by a condenser is equal to the product of its capacity multiplied by its potential difference, and as the potential differences to which C_1 and C_2 were charged were proportional to R_1 and R_2, then, by Ohm's law, when the charges on C_1 and C_2 are equal we must have the relation that

$$C_1 R_1 = C_2 R_2,$$

or

$$C_1 = C_2 \frac{R_2}{R_1}$$

If condenser C_1 absorbs part of its charge, its total charge will increase on charging for a longer time. If the charges are allowed to mix for a longer time there is more opportunity for the absorbed charge to be given up. It follows that if the time of charging is short, the capacity of the C_1 will appear to be smaller than if a longer time of charging were allowed. With good mica condensers little effect of absorption will be found. With most other dielectrics the absorption is quite marked.

Example. If in the bridge method or in the method of mixtures, $C_1 = 0.5$ microfarads, $R_1 = 2{,}340$ ohms, and $R_2 = 1{,}000$ ohms, what is C_2? Ans. $C_2 = 1.17$ microfarads.

Absolute Method. If a condenser is rapidly charged through a galvanometer and then discharged by short circuiting the condenser, the deflection of the galvanometer will be the same as though an equal charge had passed through the galvanometer in the form of a steady current during the same time. The difficulty with the method is that the galvanometer obstructs the complete charge of the condenser when the charges become very frequent. To get around this difficulty a second circuit to the galvanometer is arranged to carry a steady current equal in value but opposite in direction to the pulsating current due to the charge of the condenser. The result is that the galvanometer carries only the difference of these two currents, and when a balance is obtained, the resultant current is a small alternating current, alternately helped and hindered by the resistance and inductance of the galvanometer. The method then becomes somewhat like the Wheatstone's bridge method.

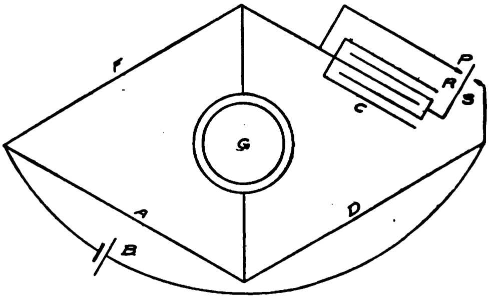

Fig. 46. Diagram for Absolute Method of Capacity Measurement.

To regulate the number of charges to a uniform number per second, a small motor running at a constant known speed or an electrically driven tuning fork of known frequency of vibration, may be used. The apparatus is arranged as in Fig. 46. If the condenser of capacity C is charged when the movable piece P makes contact with S, part of the charge will pass through the galvanometer of resistance G. The condenser is discharged when P makes contact with R. The battery B tends to send a steady current through the divided bridge circuit, part passing through F and G (both of fairly high resistance) and part through A (of low resistance). The resistance in the battery arm must be made very low in comparison with F, D, and G. Let us suppose D a fixed resistance, say, 1,000 ohms, and A, 1 ohm. In this case two proportional arms of a *postoffice* box may be used. Adjust F until a balance is obtained, with no deflection of the galvanometer. Let n be the number of charges of the condenser per second. We then have the closely approximate relation

$$C = \frac{A\ (F + G) \times 10^6}{n\,F\,(DG + DF + AG)}$$

The capacity C is in microfarads. If the factor 10^6 is omitted, the formula will give C in farads.

If the resistances in the battery branch and in A are not small, it is necessary to use a more complicated formula.

Example. When a balance is obtained $A = 1$ ohm, $F = 2{,}340$ ohms, $G = 10{,}000$ ohms, $D = 1{,}000$ ohms, and $n = 32$ periods per second. What is the capacity of C? Ans. $C = 0.01334$ microfarad.

Alternating-Current Method. If a circuit through which an alternating current is flowing includes a condenser, the charge and discharge of the condenser is repeated with every alternation of the current. The quantity of each charge is equal to the capacity multiplied by the p. d. to which the condenser is charged. The rate of charge or discharge is the value of the current at any particular instant. It is proved in treatises on alternating currents that the effective value of a current or an e. m. f. is the square root of the average square of its instantaneous values. Alternating-current ammeters and voltmeters such as those shown in Figs. 47, 52, and 53, calibrated with direct currents, show the effective values of A. C. currents and electromotive forces. The theory of alternating currents shows that the current passing in and out of a condenser, if the e. m. f. follows a sine law, is

$$I = \frac{2\,\pi\,n\,C\,E}{1{,}000{,}000}$$

From this it follows that

$$C = \frac{1{,}000{,}000\ I}{6.2832\ n\ E} = 159{,}155\ \frac{I}{n\,E},$$

when I is the effective value of the current in amperes, E the effective value of the e. m. f. in volts, n the number of cycles per second of the e. m. f. (and consequently of the current too), and C the capacity of the condenser in microfarads. If the capacity is rated in farads, omit the factor 1,000,000. In Fig. 48 the current I flows through the ammeter A, and the condenser of capacity C in series. A high resistance voltmeter V measures the potential difference E at the terminals of the condenser.

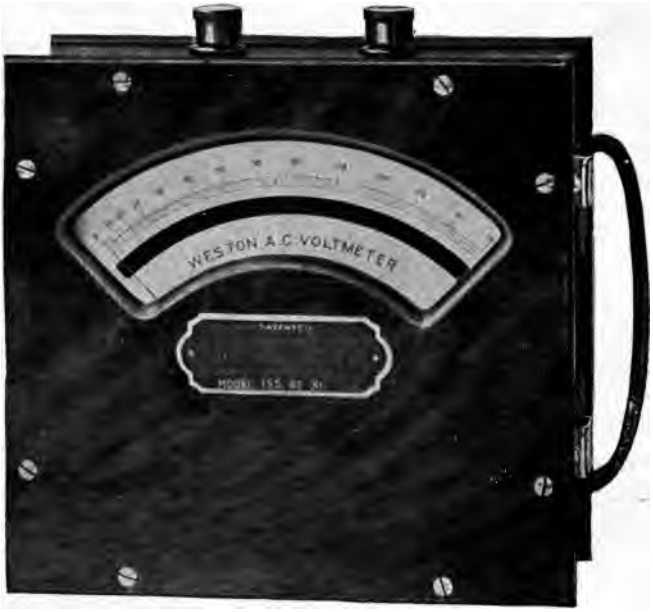

Fig. 47. Commercial Form of Portable A. C. Voltmeter.

Example. The ammeter reads 1 ampere, the voltmeter 220 volts, the frequency n is 60 cycles per second. What is the capacity of the condenser?

Ans. $C = 12.06$ microfarads.

If the voltmeter takes much current in proportion to the whole, a correction must be made for it. From the theory of alternating currents a condenser takes a current one-quarter of a period in phase *ahead* of its potential difference. A circuit in which there is both resistance and inductance takes a current lagging *behind* the potential difference. As voltmeters have some inductance, which should be small in comparison with the resistance, it will be seen that the current in the voltmeter lags a little over a quarter period in phase behind the condenser current. The relation of the currents with apparatus set up as in the previous figure is shown in Fig. 49, in which I_a, I_c, and I_v, are the currents in the ammeter, condenser, and voltmeter respectively. I_a and I_c are practically of equal length. The directions of the arrows show the phase relations.

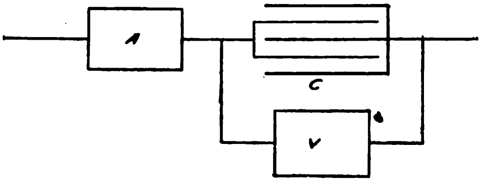

Fig. 48. Diagram for A. C. Method of Capacity Measurement.

If the condenser current is small and the voltmeter current large relatively, it will be seen from Fig. 50 that I_a may be materially larger than I_c, and that the ammeter reading must be corrected. It may be shown by trigonometry that if the voltmeter current lags by an angle a behind its potential difference, we will have

$$I_c = I_v \sin a + \sqrt{I_a^2 - (I_v \cos a)^2}$$

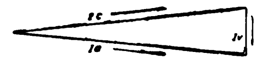

Fig. 49.

By the theory of alternating currents $\tan a = \frac{2 \pi n L}{R}$, where n is the frequency of the system, L the self-inductance, and R the resistance of the part of the circuit considered. If n, L, and R are known, a may be found.

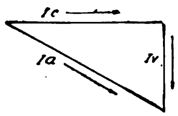

Fig. 50

Example. What correction, if any, should be made in the previous example, $I_a = 1$ amp., $E = 220$ volts, $n = 60$ cycles per second, if the voltmeter takes a current 0.06 amp. lagging 1° behind its potential difference?

Solution.

$$\sin 1° = 0.01745, \cos 1° = 0.99985$$

$$I_c = 0.00105 + \sqrt{0.9964012} = 0.00105 + 0.99820$$
$$= 0.99925 \text{ amp.}$$

As the correction of the current is far below the accuracy with which the ammeter may be read, no correction in the computed capacity of the condenser should be made.

Example. If the current observed was 0.1 amp., the other data remaining the same as in the previous example, what correction, if any, should be made?

Solution.

$$
\begin{aligned}
I_c &= 0.00105 + \sqrt{0.0064012} \\
&= 0.00105 + 0.08001 = 0.08106 \text{ amp.}
\end{aligned}
$$

The uncorrected formula gives $C = 1.206$ microfarads

The corrected formula gives $C = \underline{0.978}$ "

The difference is 0.228 "

which is 19%, a difference much too large to be ignored.

MEASUREMENT OF SELF-INDUCTANCE

Most methods of measuring inductance are too difficult for the readers of this book. The difficulty is due to the fact that a coil of wire which has inductance, has resistance also. During the increase of a current the inductance acts as a false resistance which makes the resistance appear too high. During the current's decrease, however, the inductance acts as a negative resistance, which makes the resistance appear too low. In the case of an alternating current the effect is to make the apparent resistance, called the *impedance,* higher than the real resistance. Algebraically expressed we have

$$\text{Impedance} = \sqrt{R^2 + 4\,\pi^2\,n^2\,L^2} = \frac{E}{I},$$

in which R is the real resistance in ohms, $\pi = 3.1416$, n the frequency in cycles per second, L the inductance in henrys, E the e. m. f. in volts, and I the current in amperes. The impedance of a coil is therefore not a constant quantity if R and L are constant, but depends on the frequency n. For commercial lighting n is usually 60, and for power circuits 25 cycles per second. Coils which have an iron core do not have a constant self-inductance, for the latter, with increase of current, rises slightly to a maximum and then falls off greatly for large values of the current.

Alternating-Current Method. If in a circuit, Fig. 51, the current through a coil of known resistance R and unknown inductance L, is measured by means of an ammeter A (two views of a well-known commercial instrument are shown in Figs. 52 and 53), and the potential difference E over the coil is measured by a voltmeter V, we have the following relation

$$2 \pi n L = \sqrt{\frac{E^2}{I^2} - R^2},$$

or

$$L = \frac{1}{2 \pi n} \sqrt{\frac{E^2}{I^2} - R^2}$$

Example. In a circuit for which the frequency $n = 60$ cycles per second, $R = 0.1$ ohm, $E = 110$ volts, and $I = 10$ amperes, what is the value of the inductance L?

Ans. $L = 0.0292$ henry.

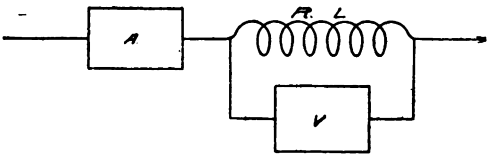

Fig. 51. Diagram for A. C. Method of Self-Inductance Measurement.

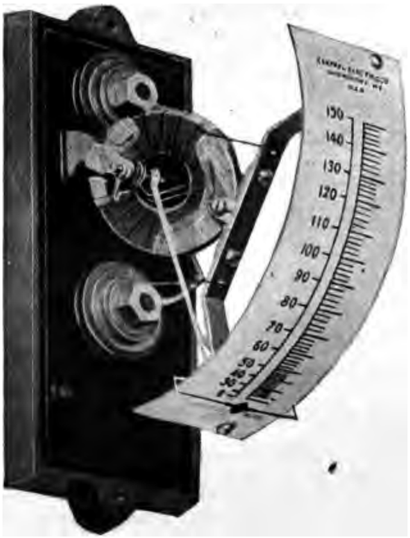

Fig. 52. Thomson Inclined Coil A. C. Ammeter.

If the resistance or inductance is very high, so that the total current is small, a correction must be made for the portion of the current passing through the voltmeter. The corrected formula is very complicated when expressed directly in terms of L, l, R, r, the inductances and resistances of the coil and the voltmeter respectively, n the frequency, E the potential difference, and I_a the current through the ammeter. If the tangents of the lag of the current in the two branches of the circuit are for the coil

$$\tan b = \frac{2 \pi n L}{R},$$

and for the voltmeter circuit

$$\tan a = \frac{2 \pi n l}{r},$$

we get the equation for the cosine of the difference between a and b

$$\cos\left(\pm(a-b)\right)=\frac{rR}{2}\left(\frac{I_a^2}{E^2}-\frac{1}{r^2}-\frac{1}{R^2}\right)$$

Usually *b* is greater than *a*, so we write the formula

$$\cos(b-a)=\frac{rR}{2}\left(\frac{I_a^2}{E^2}-\frac{1}{r^2}-\frac{1}{R^2}\right)$$

As the constants of the voltmeter are supposed to be known, the angle *a* is known; therefore *b* may be determined and we finally get

$$L=\frac{R\tan b}{2\pi n}$$

If *b* is so small that the difference between *a* and *b* is not greater than *a*, there is always the possibility that the previous formula is the cosine of $(a-b)$. To determine which result to take, it is necessary to repeat the experiment with additional resistance in one of the two branches, and to take the value of *L* that equals one of the previous solutions. As a rule, however, the difference between *a* and *b* will be greater than *a*, and the incorrect result will lead to a negative value for *b*. As *b* must be positive, the negative result is rejected as impossible.

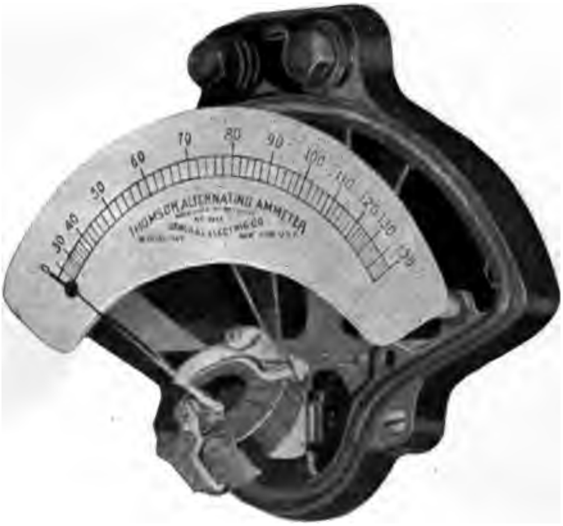

Fig. 53. Thomson Inclined Coil A. C. Ammeter.

Bridge Method. Two self-inductances may be compared by a modification of the Wheatstone's bridge method. In the simplest of many bridge methods the coil of unknown inductance X is one arm of the bridge; a double coil, Fig. 54, whose inductance L may be varied by rotating one part to various positions inside the other part, and whose inductance is known for each position and marked by a pointer on a circular scale for each position, is inserted in an adjacent arm. The other arms are non-inductive resistances R_1 and R_2. The arrangement is shown in Fig. 55. The inductive branches have certain resistances R_3 and R_4. A regular Wheatstone's bridge balance is obtained by adjusting the resistances R_1 and R_2, care being taken to close the key K_2 in the galvanometer branch several seconds after closing the key K_1 in the battery branch. This precaution is necessary to insure that the currents are steady when the galvanometer key is closed, as inductive effects are produced only when the current is changing in value. When a balance is obtained $R_1 : R_2 :: R_3 : R_4$. If now the galvanometer key K_2 is closed first, the false resistance due to inductance, will cause the galvanometer to deflect when the battery circuit is closed. After oscillating a number of times the galvanometer finally comes to rest at its zero position. On opening the battery circuit the first deflection is in the opposite direction. If, now, the variable inductance is adjusted until there is no deflection on closing the battery circuit, the galvanometer circuit having been closed in advance, the false resistances due to inductance must increase the apparent resistances in both inductive branches in proportion to their real resistances, from which it follows that

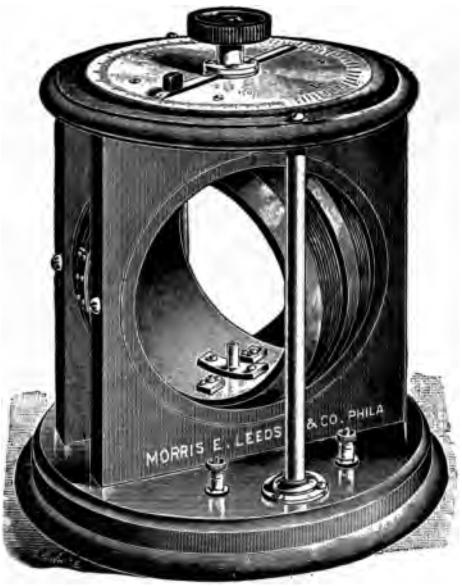

Fig. 54. Variable Self-Inductance.

$$X : L :: R_3 : R_4 :: R_1 : R_2,$$

and we get the relation

$$X = L\frac{R_1}{R_2}$$

If no variable standard of inductance is available, there are various modifications of the bridge method which may be used. These methods are as a rule very complicated and consequently beyond the scope of this course.

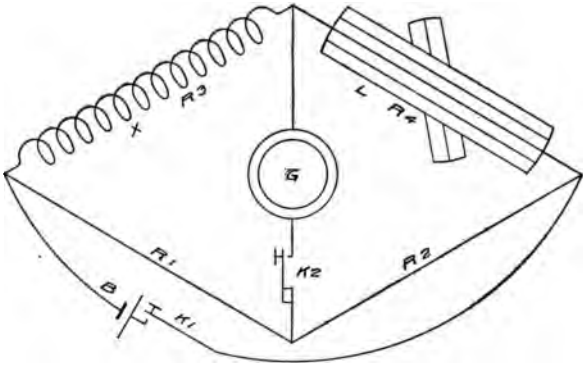

Fig. 55. Diagram of Bridge Method of Self-Inductance Measurement.

Condenser Method. The self-inductance of a coil may be compared with the capacity of a condenser. The bridge is set up as indicated in Fig. 56. The condenser of capacity C is in parallel with M which is one part of a constant resistance R_1. $R_1 = M + N$. R_2 and R_3 are resistances, one or both of which may be varied at will. The coil of resistance R_4, whose inductance L is to be measured, is in the fourth arm of the bridge. The galvanometer G, battery B, and keys K_1 and K_2 are as usual. The resistances are adjusted in R_2 and R_3 until a balance is reached, when K_2 is closed several seconds after closing K_1.

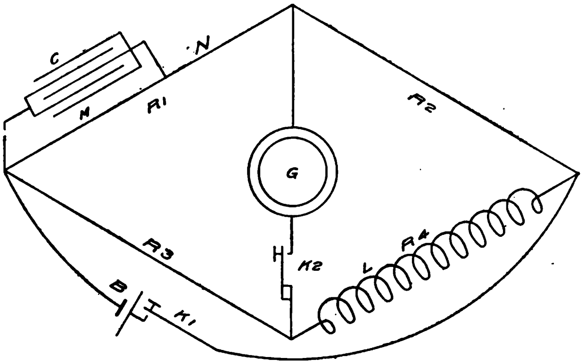

Fig. 56. Diagram of Bridge Method of Self-Inductance Measurement.

Then $R_1 : R_2 :: R_3 : R_4$, the usual Wheatstone's bridge relation. The galvanometer key K_2 is next kept closed and resistance shifted between M and N, care being taken to keep $M + N = R_1$ constant, until no deflection is produced at the instant of closing K_1. The explanation of the method is quite complicated, but leads to the simple result,

$$L = \frac{C\ R_2\ R_3\ M^2}{R_1^2\ 10^6} \text{ henries.}$$

C is in microfarads, R_1, R_2, R_3, and M in ohms, and the result for L in henries.

Example. The resistance R_1 was kept constant at 1,000 ohms, part of which M, when a balance was reached, was 516 ohms. $R_2 = 1{,}000$ ohms, $R_3 = 1{,}260$ ohms, and $C = 1$ m. f., makes up the balance of the data. What was L?

Ans. $L = 0.3355$ henry.

MEASUREMENT OF MUTUAL INDUCTANCE

If two electric circuits are in the neighborhood of one another, the increase or decrease of the current of one will produce a change in the magnetic field which will act to produce an e. m. f., and consequently a current in the second circuit if it is closed. If the current in one circuit varies at the rate of one ampere increase or decrease per second and an e. m. f. of one volt is produced in the second circuit, we say the mutual inductance is one henry. If the mutual inductance is constant, the e. m. f. produced in the second circuit is proportional to the rate of change of current in the first; also larger inductances produce proportionately larger e. m. f.'s in the second circuit with equal rates of change of current in the first. Algebraically expressed, if I_1 and I_2 represent any two values of the current, supposed to be increasing at a uniform rate, and t is the interval of time between these values of the current, and if E is the e. m. f. produced in the second coil whose mutual inductance with respect to the first is M, then

$$M\,\frac{I_2 - I_1}{t} = E$$

The inductance between the coils is called *mutual* because a certain increase of current in either will produce the same e. m. f. in the other regardless of which circuit has the original current. The relation is therefore *mutual.* If the circuit has an iron core the mutual inductance is not strictly constant, but with increasing current increases to a maximum and then falls again. As it is very difficult, if not impossible, to control the precise rate of change of the current so that it will increase at a uniform rate, either of two methods may be used to obtain the value of M without keeping the rate of change of I constant.

Ballistic Galvanometer Method. In this method the second circuit, called the *secondary* circuit, is of known resistance R_2, and includes a ballistic galvanometer G and sufficient extra non-inductive resistance T_2 to control the deflection within reasonable bounds. The other circuit, called the *primary* circuit, includes a key K, an adjustable resistance r_1, a battery B, and an ammeter A. The arrangement is shown in Fig. 57. The primary coil is P and the secondary S.

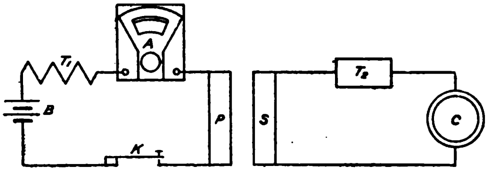

Fig. 57. Diagram of Ballistic Method of Mutual Inductance Measurement.

On closing the key K the current in P begins to increase and the galvanometer G begins to deflect. If we make no allowance for the false resistance due to the self-inductance of the secondary circuit, the current I_2 in the secondary is $I_2 = \frac{E}{R_2}$. As the current I_1 in the primary requires a considerable part of a second nearly to reach its maximum or steady value, it is evident that the secondary current I_2 will require at least an equal time to rise from zero to its maximum and to fall nearly to zero again. Theoretically it takes an infinite time for this to happen, but if the total resistance R_1 in the primary and R_2 in the secondary are reasonably large in comparison with their self-inductance, the time practically necessary is a second or so. The current I_2 in the secondary will, therefore, during the times that it flows, cause a certain quantity Q of electricity to pass through the galvanometer. If the ballistic galvanometer has a long period of swing—a condition required of ballistic galvanometers—practically the whole of the quantity Q will pass before the galvanometer gets far from its zero position. As the e. m. f. of the secondary is proportional to the rate of increase of the primary current I_1, it is evident that the *total* quantity flowing in the secondary will depend on the *total* rise of current in the primary in precisely the same way as the current *at any instant* in the secondary depends on the *rate of increase* of the current in the primary. As before mentioned, we have ignored the false resistance due to the self-inductance of the secondary. We have learned, however, in the chapter on self-inductance, that during the rise of a current the self-inductance increases the apparent resistance, but during its fall the opposite effect is produced. Therefore, during the rise and fall of the current in the secondary, no appreciable error is caused by ignoring the self-inductance of the secondary. The final result is $M I_1 = Q R_2$, or

$$M = \frac{Q R_2}{I_1}$$

To get the value of Q we must know the relation between the throw d_2 of the ballistic galvanometer to the quantity of electricity producing the throw. This may be found by charging a standard condenser of known capacity C by a standard cell of known e. m. f. E_s, and noting the deflection d on discharging the condenser through the ballistic galvanometer. As the charge of the condenser is $E_s C$, we obtain the result

$$Q : d_2 : : E_s C : d,$$

or

$$Q = \frac{d_2 E_s C}{d}$$

Substituting in the earlier equation the value of Q and expressing the capacity of the condenser in microfarads,

$$M = \frac{d_2 E_s C R_2}{d I_1 \times 10^6}$$

Example. The current I_1 in the primary coil rises on closing the circuit to 1 ampere. The total resistance in the secondary circuit is 10,000 ohms. The deflection produced is 21.3 cm. With a Weston normal cell of 1.0184 volts and a condenser of 0.2 microfarads, a deflection of 23.0 cm. was produced on discharging the condenser through the galvanometer. What is the mutual inductance M?

Ans. $M = 0.001886$ henry.

Alternating-Current Method. The previous method may be varied by putting in the secondary an alternating-current voltmeter of high resistance, and in the primary an A. C. ammeter, an e. m. f. source of sine form, and whatever resistance is needed to control the current. From the theory of alternating currents, the e. m. f. produced in the secondary is

$$E_2 = 2 \pi n M I_1,$$

or

$$M = \frac{E_2}{2 \pi n I_1},$$

when n is the frequency in cycles per second, M the mutual inductance, and I_1 the primary current. This relation assumes that the current in the secondary is too small to affect the flux of magnetic lines crossing over from primary to secondary. If the resistance of the secondary coil is small in comparison with that of the voltmeter, the voltmeter reading E_v is taken as E_2. If the coil is of too high resistance to make the last assumption allowable, the reading must be multiplied by $\frac{R_2}{R_v}$, when R_2 is the total resistance of the secondary and R_v the resistance of the voltmeter. We then get

$$E_2 = \frac{R_2}{R_v} E_v = 2 \pi n M I_1,$$

or

$$M = \frac{R_2 E_v}{2 \pi n R_v I_1}$$

If the secondary current I_2 is too large for its magnetic effect to be ignored, or if the current in the primary does not follow a simple sine law, the problem becomes too complex for easy solution.

Example. The current I_1 in the primary is 1.1 amperes, the frequency n is 60 cycles per second, the resistance of the secondary coil is negligible. The reading of the voltmeter E_v is 110 volts. What is the mutual inductance M? Ans. $M = 0.2653$.

Carey-Foster Method. Let a battery of constant e.m.f. be connected in series with one of the two coils P whose mutual inductance is to be determined, a known resistance R_1, and a key K, Fig. 58. Let the ballistic galvanometer G and another resistance R_2 be connected in series with the other coil S. Then if I be the steady current produced by the battery B through P, and M be the mutual inductance, and r be the resistance of the circuit through S, R_2, and the galvanometer, then the quantity of electricity Q, passing through the galvanometer on closing or opening the circuit will be

$$Q_1 = \frac{MI}{r}$$

Next, if the galvanometer be removed from this circuit and put in series with a condenser whose capacity is C, which is connected as a *shunt* to the resistance R_1, on opening or closing the battery circuit the quantity of electricity

$$Q_2 = IR_2C$$

By combining these two equations it is possible to find the relative values of C and M. In practice it is much more desirable to combine these two circuits, as shown above, so that the charge and discharge of the condenser and the currents produced at the same time in S by mutual induction are in the same direction through C, R_2, and S. Then if the resistance R_1 and R_2 and the capacity C are adjusted until no deflection of the galvanometer is produced, the following may be written:

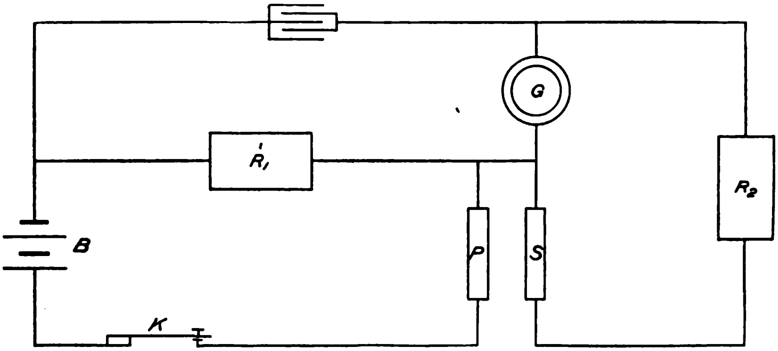

Fig. 58. Diagram of Carey-Foster Method of Measuring Mutual Inductance.

$$Q_1r = MI, \text{ but } Q_1 = IR_1C; \text{ hence } M = CR_1r$$

Attention is called to the fact that in order that the galvanometer current may be 0 at every instant during the establishment of the steady current, it is essential that the coefficient of self induction of the coil S should be equal to the coefficient of mutual induction. Under this condition it is possible to replace the galvanometer by a telephone.

Example. Small Induction Coil, no iron core. Resistance of secondary, 194 ohms. Capacity of condenser, 4.9.26 microfarads

The secondary coil could slide endways remaining coaxial with the primary. The following are the results with the centers of the two coils as nearly coincident as possible:

R_1	r	R_1r (C.G.S. Units)	R_1	r	R_1r (C.G.S. Units)
15	194 + 217	6165×10^{18}	10	194 + 423	6170×10^{18}
14	+ 247	6174	9	+ 490	6156
13	+ 282	6188	8	+ 576	6160
12	+ 322	6192	7	+ 688	6174
11	+ 367	6171	6	+ 835	6174

$$\text{Mean value of } \frac{M}{C} = 6172.4 \times 10^{18}$$

Hence

$$M = 4.926 \times 10^{-15} \times 6172 \times 10^{18} = 3.04 \times 10^{7} \text{ or } .0304 \text{ henrys.}$$

MAGNETIC MEASUREMENTS

Certain materials, notably iron (or steel), nickel, and cobalt, have a property known as *magnetism*. These materials when magnetized have the property of attracting soft iron. The modern view of magnetism is that it is a property of the individual molecules of a body. A body which seems to be unmagnetized probably has its molecules arranged in more or less irregularly formed closed chains, Fig. 59, which produce no outside effect. To magnetize a body, it is, according to this theory, necessary to break the chains and to rearrange the connections of the molecules so that the ends of the chains of molecules come out at points on the surface where so-called magnetic charges appear, Fig. 60. The centers of action of these chain ends are called *poles*. In the simplest magnets there are two poles, and if the magnet is free to turn in a horizontal plane, one of the poles is turned toward the north and the other toward the south approximately. In general, the magnetic meridian, determined by the line joining the poles when at equilibrium in the horizontal plane, does not agree exactly with the geographical meridian. The line of no

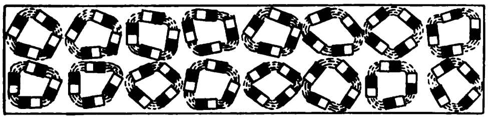

Fig. 59. Supposed Molecular Condition of a Piece of Unmagnetized Steel.

variation at present is located near the eastern shore of Lake Michigan and is moving westward. A century ago it was near the eastern end of Lake Erie. For points to the east of the line of no variation, the magnetic compass points west of north; and for points west of this line of no variation, it points east of north. The north seeking pole of a magnet is commonly called the *north pole* and the other the *south pole* of the magnet. If the magnet is free to turn in all directions, the north pole will dip downward and the south pole rise upward for points in the northern hemisphere and *vice versâ* for points in the southern hemisphere. The dip in Chicago is in the neighborhood of 70°.

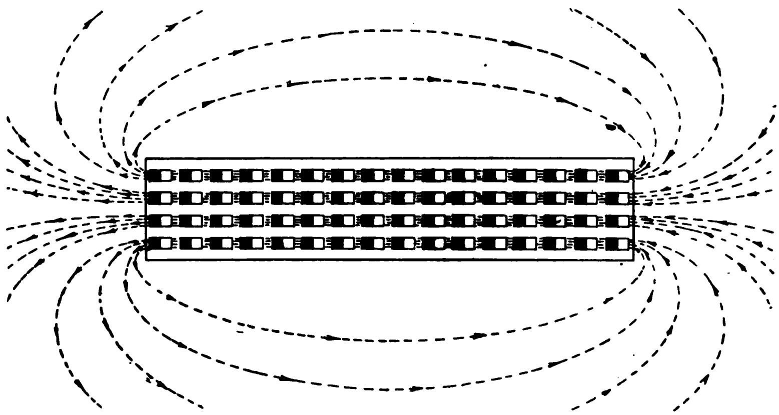

Fig. 60. Supposed Molecular Condition of a Magnetized Piece of Steel.

From the above it appears that the earth has a magnetic field, meaning by magnetic field an extent of space where magnetic forces are to be found. If a magnet with poles 1 cm. apart and of unit strength is in a unit magnetic field, it will act on each pole with a force of one dyne; and if the poles are turned so that the directions of the forces are at right angles to the line connecting the poles, a unit turning moment, or torque, is exerted on the magnet. In the first chapter of this book, unit magnetic pole was defined as a pole which, at a distance of one centimeter in air from a like pole, produces a repulsion of one dyne of force. It appears then that a magnetic field has both direction and magnitude, and is what is called a *vector* quantity. To define a vector quantity both magnitude and direction must be known.

Methods of Magnetizing. Besides natural magnets, composed of the magnetic oxide of iron and known as *loadstones,* artificial magnets may be made by subjecting hard steel to a magnetic field. The magnetic field used may be due to a loadstone, or an artificial magnet previously made, or a magnetic field due to an electric current in a coil of wire. Before 1819, when Oersted discovered the magnetic field produced by a current, the source of all artificial magnets was directly or indirectly the loadstone. Permanent magnets nowadays are practically all magnetized by means of electric currents.

Lines of Force and Permeability. To Michael Faraday we owe the notion of lines of force to express the vector quantity defining the magnetization. The direction of the lines is used to indicate the direction of magnetization, and the number of lines per square centimeter indicates the magnitude of the magnetization. The numbers of lines per square centimeter is commonly called the *flux density* or *flux of force* per square centimeter. The unit value occurs when there is one line per square centimeter, and is called the *gauss.* The symbol B is used to express this quantity algebraically. As different materials when put in equal fields take different degrees of magnetization, the relation of field strength to flux, known as the *permeability,* must be known. If H indicates the field strength, B the flux per sq. cm., and μ the permeability, we have the relation

$$\mathrm{B} = \mu \, \mathrm{H}$$

When the section considered has an area A sq. cms. and the average flux intensity is B, the total flux designated by Φ is algebraically expressed as

$$\Phi = \mathrm{B}\,A = \mu\,\mathrm{H}\,A$$

The unit in which Φ is measured is the Maxwell and is equal to one line of force.

If the lines of force pass from one material into another of different permeability μ, the lines representing B and Φ will usually suddenly change direction at the surface of separation between the materials, commonly called *media,* except in the case that the lines are normal to the surface. If the permeabilities of two media are μ_1 and

μ_2, and the angles between the normal and the lines in the two media are a_1 and a_2, we have the relation

$$\frac{\tan a_1}{\tan a_2} = \frac{\mu_1}{\mu_2}$$

Lines of force follow by preference paths of high permeability, though, other things being equal, they tend to follow the shorter paths. Consequently it rarely happens that the lines are straight, as they converge toward spaces filled with bodies of high permeability and there diverge again in spaces of low permeability. In general the total flux Φ distributes itself so that, length of path and permeability considered, it takes the easiest course. A line of force never ends, but always returns on itself. Many writers carelessly confuse and use the same unit (the gauss) for measuring B and H, for both are vector quantities and may be represented by lines. To avoid confusion we shall not represent H by lines. For highly magnetic materials, B is much greater than H. In the case of iron the ratio may be as high as $\mu = 3{,}000$ for moderate values of B. The value of μ is not constant for the same material for different values of B. In the case of soft iron the permeability for low values of B may be about $\mu = 120$, rising to $\mu = 2{,}000$, or in good samples $\mu = 3{,}000$, when B reaches a value between $B = 5{,}000$ to $B = 8{,}500$. For nickel and cobalt the highest value of the permeability is about $\mu = 200$. As a standard of comparison the permeability of air is taken as $\mu = 1$ and is believed to remain sensibly constant for all values of B. It follows that in air $H = B$ (numerically). Materials more magnetic than air, for which $\mu > 1$, are called *paramagnetic*. Materials less magnetic than air, for which $\mu < 1$, are called *diamagnetic*. No magnetic material is without permeability, that is $\mu = 0$; in fact, even the most diamagnetic material, bismuth, is within a fraction of one per cent as permeable as air. Magnetic insulation is therefore impossible and to avoid magnetic leakage of lines of force from a prearranged path, it is necessary to distribute the *magnetomotive force* over the whole path.

Magnetomotive Force. By analogy with electric circuits, where the potential difference over each unit length of the circuit is found by multiplying the current by the resistance of that unit length, or dividing the current by the conductivity (the reciprocal of resistance) for the unit length, we see that in a part of a magnetic circuit, one centimeter long and one square centimeter in section, μ is analogous to the conductivity per cm. cube of an electric circuit. **B** is analogous to the current per sq. cm. of section of the conductor and **H** is analogous to the potential difference per centimeter length. **H** is called the magnetomotive force per unit length, and the total magnetomotive force is the average value of **H** multiplied by the length of the circuit l. It follows that the magnetomotive force (m. m. f.) is

$$\text{m. m. f.} = \mathbf{H}\, l$$

If the magnetic field is produced by a current in a wire, the intensity of the field is greatest at the surface of the wire and falls in value to zero at the middle of the wire, and outside the wire falls in value according to the law of the reciprocal of the distance from the center. If, however, the wire is coiled into a long, straight coil of uniform section, called a *solenoid*, the magnetic field for the portion far from the ends is practically zero outside the solenoid and of uniform value inside the solenoid. If there are n turns of wire per centimeter and a current of I amperes, the inside magnetic field is

$$\mathbf{H} = 0.4\, \pi\, n\, I = 1.2566\, n\, I$$

If the whole number of turns of wire is N, the magnetomotive force is

$$\text{m. m. f.} = 1.2566\, N\, I$$

The unit of m. m. f. is the *gilbert*, or that value of magnetic force which will establish one line or one maxwell per centimeter cube of air. Many practical authorities prefer to express the m. m. f. in *ampere turns* $N\,I$ omitting the factor 1.2566. This leads to some confusion if the fact is not made clear by stating that the m. m. f. is in ampere turns. If a long solenoid is bent into the form of a ring, it is called a *ring* solenoid. If the width of the ring is small in comparison with its diameter, it is assumed that the average value of **H** is equal to its value along the central line of the ring.

Reluctance. If a circuit is l centimeters long and averages A sq. cm. in section, the reluctance **R** of the whole circuit is

$$\mathbf{R} = \frac{l}{A\, \mu}$$

The unit in which reluctance is measured is the *oersted*.

By analogy with Ohm's law the magnetic flux Φ is

$$\Phi = \frac{\text{m. m. f.}}{\text{reluctance}} = \frac{\text{m. m. f. } A\,\mu}{l} = \mathrm{B}\,A = \mu\,\mathrm{H}\,A$$

Hysteresis. When iron or steel has been magnetized and the magnetizing force removed, a portion of the magnetization will still remain as more or less *permanent* magnetization. If next the magnetizing force is applied in the reverse direction, the magnetization will not be reversed until the m. m. f. has reached a certain value, *i. e.*, until H reaches a certain value. The residual value of B when the field is reduced to zero is called the *remanence* or *retentiveness* by some writers. The reverse field, m. m. f., necessary to reduce B to zero is called by the barbarous term (as Professor Mascart puts it) of *coercive force.* Further increase of the reverse m. m. f. will cause a rapid rise of B. With repeated cycles of change between positive and negative m. m. f.'s the values of H and B go through cycles. The tendency of B to lag behind H is called *magnetic lag* or *magnetic hysteresis,* hysteresis being the Greek word for lagging behind. A certain amount of energy is expended in the cycle and appears in the form of heat generated in the iron. This loss of energy per cycle is repeated *n* times per second, and the power used is known as the *hysteresis loss,* and is measured in watts per cubic centimeter. Sometimes the energy lost per cycle is measured in joules. One joule equals 10,000,000 ergs. The relation of H and B during the hysteresis cycle may be expressed in the form of a curve plotted in terms of H (horizontally) and B (vertically). The curve shown in Fig.

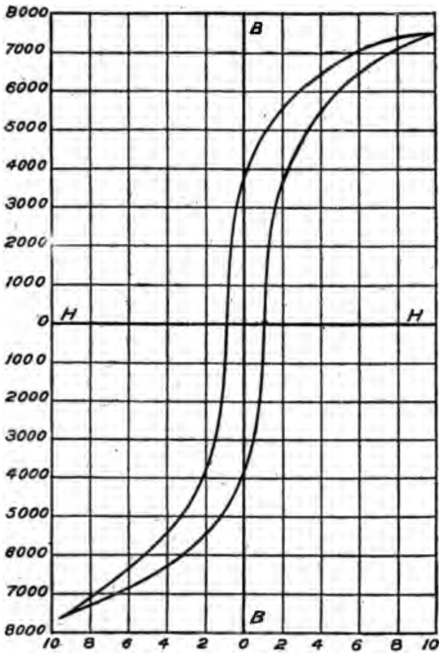

Fig. 61. Hysteresis Diagram.

61 is known as a *hysteresis curve.* If H and B are plotted to scale, the area of the curve divided by 40,000,000 π, or 125,660,000, gives the energy lost per cycle in joules. For measurements of hysteretic losses the sample may be made into the form of a ring with small difference between its largest and smallest radius. The ring may then be wound with a coil of wire in the form of a ring solenoid and the current and turns in the coil will determine the m. m. f. and consequently H in the core.

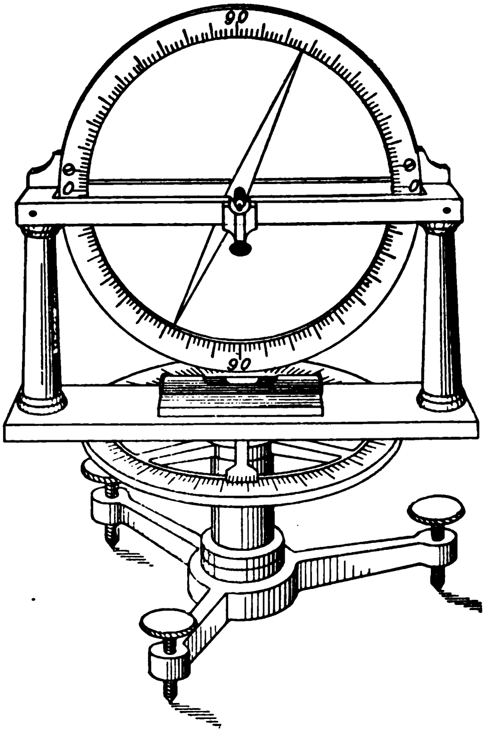

Fig. 62. Magnetic Dip Needle.

Magnetic Dip. *By Dip Needle.* If a long and slender magnetic needle, Fig. 62, with pointed ends is mounted on an axis passing precisely through its center of gravity, at the middle of an accurately graduated circle standing vertically in the magnetic meridian, the north pole of the needle will point downward from the horizontal by an angle equal to the magnetic dip. The angle of dip may then be read directly from the circle. As, however, it is difficult to magnetize the needle with exact uniformity, the bearing should be made reversible so that the needle may be turned over. Any irregularity of magnetization may be eliminated by taking the mean of the two readings of the magnet in the two positions. In case the bearing fails to pass exactly through the center of gravity, the error may be compensated by reversing the magnetization of the needle, care being taken to use the same magnetic field as before, and then repeat the observations of the dip with the magnet in both positions. The mean of the four readings will give the value of the dip. If the divided circle is not in the magnetic meridian, an error will be caused which is slight for slight deviations from the meridian. A compass needle mounted over the divided circle will locate the magnetic meridian well enough for practical purposes.

Earth Inductor Method. Another method is by use of the earth inductor which is a coil of wire mounted in a frame and which may be turned about an axis in the plane of the coil. No iron or other magnetic material should be used in the apparatus. The frame is carried on a support by which the axis of rotation may be made vertical or horizontal. The apparatus is shown in Fig. 63. All heavy parts are made of brass which is practically non-magnetic. If the coil is turned about a vertical axis, as shown in the figure, from an east and west plane through 180° to the reverse position in the same plane, the number of lines = B A cos δ of the earth's magnetic field passing through the coil, will be cut by the coil, and if a ballistic galvanometer is in the circuit, its deflection d_1 will be proportional to the flux cut. If now the axis of rotation be made horizontal and the coil horizontal, a reversal in position will cut the number of lines = B A sin δ of the earth's magnetic field passing through the coil. The resulting deflection d_2 of the ballistic galvanometer is proportional to the flux cut. We then have the relation

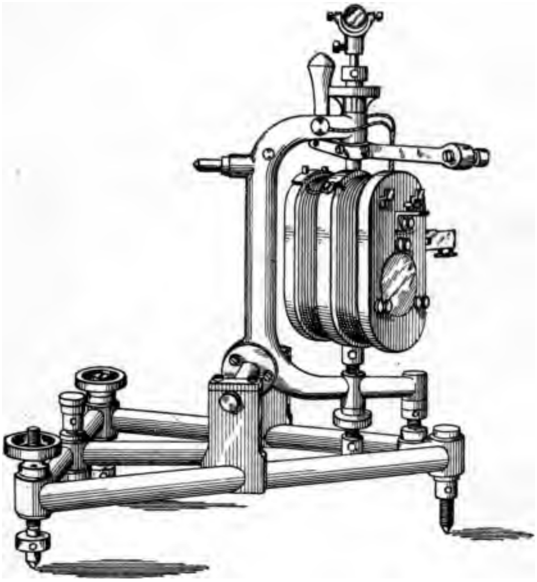

Fig. 63. Earth Inductor.

$$\tan \delta = \frac{d_2}{d_1}$$

If the deflections are small they may be increased by reversing the position of the coil on the return swing of the galvanometer, and continuing until the amplitude of swing becomes constant. The deflections in two positions of the axis of the coil are proportional to the *horizontal* and the *vertical* components respectively of the magnetic field, and the ratio of the second to the first gives the tangent of the dip of the earth's field.

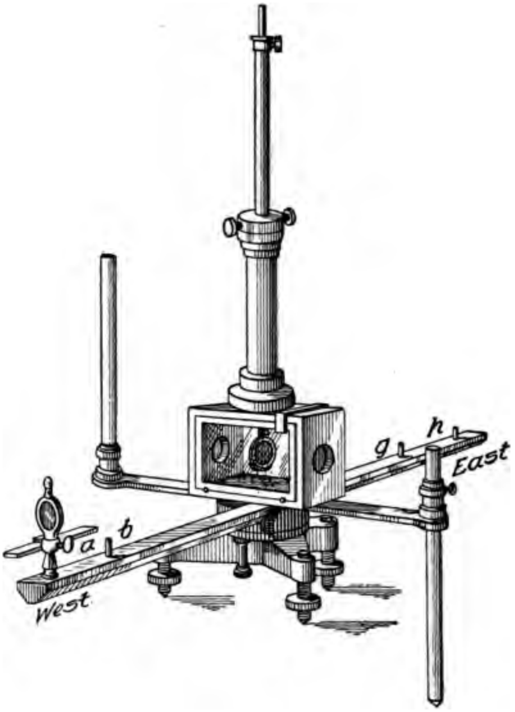

Fig. 64. Magnetometer.

The angle of dip varies from + 90° at the earth's north magnetic pole to – 90° at the south magnetic pole. It is zero at the magnetic equator.

The Earth's Magnetic Field. If in the previous method, the data of the earth inductor and of the ballistic galvanometer are known, the value of B may be determined. As the experiment is performed with air as the medium, the value of μ is 1, and H is numerically equal to B. The horizontal component of the earth's magnetic field is frequently spoken of as H, meaning thereby the *horizontal* component. In the same way V is the vertical component. The tangent of the angle of dip is the ratio

$$\frac{V}{H}$$

Magnetometer Method. The horizontal component may be measured by means of two magnets, one of which is of light weight and the other relatively large and heavy, both carrying mirrors. The little magnet and mirror may be suspended at the center of an instrument called a *magnetometer,* Fig. 64. The second magnet, of considerable size, is mounted on a support at its center at a fixed distance to the east of the little magnet and in an east and west position (point *g* of the figure). The small magnet takes up a position parallel to the resultant of the horizontal component of the earth's field and that of the large magnet. By means of a telescope and scale at a known distance, the angle of the deflection is measured. The other pole of the large magnet is now turned toward the small magnet by turning the large magnet end for end. The deflection measured by the telescope and scale should be as before, only in the opposite direction. The large magnet is now transferred to a support *b* at an equal distance to the west of the small magnet, and the observations repeated, obtaining two more deflections. All four deflections should be equal. If they differ slightly the mean is taken; if they differ much, something is wrong with the arrangement and the apparatus should be examined and the trouble corrected. The observations are now repeated at a larger distance (points *a* and *h*), and four more observations taken. From the data, assuming that the magnetic field falls off according to the inverse square of the distance from the poles of the magnet, a pair of equations may be formed from which the strength of the magnet's poles in terms of the horizontal component of the earth's field and their distance apart may be calculated. This distance apart of the poles will be found to be somewhat less than the length of the large magnet. The product of the pole strength and length between poles is called the *magnetic moment* of the magnet, represented by M. If r_2 = the larger distance, r_1 the smaller distance, and Φ_1 and Φ_2 the deflections, we get

$$\mathrm{M} = \frac{\mathrm{H}}{2} \frac{r_2^5 \tan \Phi_2 - r_1^5 \tan \Phi_1}{r_2^2 - r_1^2}$$

The large magnet is now hung up by a fine wire and set to vibrating. The period of vibration T_1 is determined by measuring with a stop watch the time of a considerable number of vibrations and finding the time of a single swing. If K_1 is the moment of inertia and θ the correction for the rigidity of the wire, we have

$$T_1 = \pi \sqrt{\frac{K_1}{\mathrm{M\,H}(1 + \theta)}}$$

As there may be some difficulty in computing the moment of inertia

K_1, it is usual to add a brass ring of rectangular section and easily computed moment of inertia K_2. This ring must be placed on top of the magnet so that its center lies on the prolongation of the suspending wire. The moments of inertia are then added to get the total moment. There are reference marks on opposite ends of a diameter of the ring and corresponding marks on portions of a circle of the same radius of the ring marked on the top surface of the magnet to ensure precise centering. With the ring in position, the new time T_2 of vibration is determined.

$$T_2 = \pi\sqrt{\frac{K_1 + K_2}{\mathrm{M\,H}(1+\theta)}}$$

θ is determined by turning the torsion head, from which the magnet is hung by the wire, through a considerable angle and determining by the telescope and scale the angle through which the magnet follows. This enables one to compute how much effect the rigidity of the wire has on the restoring force which is due principally to the earth's field. If on turning the torsion head an angle α, the magnet follows an angle β, there is obtained, $\theta = \frac{\beta}{\alpha - \beta}$. Combining the earlier equations gives

$$\mathrm{M\,H} = \frac{\pi^2 K_2}{(1+\theta)(T_2^2 - T_1^2)}$$

Substituting the value of M previously obtained we get on solving for H

$$\mathrm{H} = \pi\sqrt{\frac{2K_2(r_2^2 - r_1^2)}{(1+\theta)(T_2^2 - T_1^2)(r_2^5 \tan\Phi_2 - r_1^5 \tan\Phi_1)}}$$

The value of H in the southern part of Michigan is about 0.18 and the vertical component is about three times as strong or about 0.54. As the presence of masses of iron in the neighborhood has a considerable effect on the dip of the magnetic field and also on the value of the field, a measurement made in any place with iron masses near by, cannot be assumed as valid for even other parts of the same building. Laboratories for the study of the earth's magnetic field should have all iron excluded from the building materials and from the apparatus except the magnets.

If the angle of dip δ has been found by one of the previous methods, the total value of the earth's magnetic field F is

$$F = \frac{H}{\cos \delta}$$

The bars *N* and *L* shown in Fig. 51 are not used in this experiment.

Magnetic Flux and Permeability. There are a number of excellent methods of determining flux and permeability, of which the following will suffice for the purposes of this work.

Divided Bar Method. The divided bar method assumes that the material under test is in the form of two long iron bars or rods with the ends ground and polished into accurately plane surfaces. One bar, with the polished end upward, is mounted in a long solenoid, the polished end being at the middle of the solenoid. The other bar is placed on top of the first with the polished ends resting one on the other and accurately centered. The upper piece is attached to a spring balance which is used to measure the tension necessary to separate the bars. If the weight of the upper piece is subtracted, the remainder gives the pull. The bars and the solenoid must be long enough to have the magnetic field H at the surfaces in contact practically equal to what it would be in an infinitely long solenoid, for which *I* is the current in amperes and *n* the number of turns of wire per centimeter length, otherwise a correction must be made for the ends.

$$H = 0.4 \pi n I = 1.2566 n I$$

If the area of the ends of the bar is *S* sq. cm., and the force in grams (weight) *F*, the value of gravity *g* (= 980 about), we get for the flux density B,

$$B = \sqrt{\frac{8 \pi g F}{S}} = 156.9 \sqrt{\frac{F \text{ (grams)}}{S \text{ (sq.cm.)}}}$$

If the pull F is measured in pounds and the area *S* in square inches, we must allow for the ratio of the units. We then obtain

$$B = 1316 \sqrt{\frac{F \text{ (pounds)}}{S \text{ (sq. in.)}}}$$

In using the method, the spring balance should be supported in guides and drawn upward gradually by means of a turn-buckle or analogous means. The last reading before the bars separate is the one to be taken. If the bars are rounded at the corners an error will be made because the value of B will be increased at the smaller section to a greater value than back in the rod, as the total flux Φ is spread over a smaller area. The pull will be increased because from the above formula it appears that the pull is proportional to the product SB^2. Therefore avoid rounding the edges. If the surfaces do not fit one another, the air where they do not touch will have a lower permeability and there will be a tendency for some of the flux to escape at the side, thus reducing B and consequently the pull.

To obtain the permeability μ divide B by H. As mentioned earlier the permeability increases as B increases, reaching a maximum for moderate values of B and then falls off rapidly for further increase of B.

A magnetization curve (B, H curve) or a permeability curve (B, μ curve) may be plotted from values obtained for different values of B, H, and μ. A bar of iron which has never previously been magnetized, will behave for small values of H differently from what it will again. For this reason the values of H used should increase gradually from lower to higher values. A bar once magnetized cannot be brought back to its original condition by any process except heating it to the temperature at which it becomes practically unmagnetic and then cooling it again, retempering it if necessary. If it is demagnetized by reversing the direction of the field and reducing the latter to lower values gradually on each reversal, most of the magnetism may be removed. This is supposed to reduce the magnetization of the bar to a set of concentric magnetizations in opposite directions in successive concentric layers. This is not quite equivalent to the irregular chains of molecules in a bar which has never been magnetized. A bar which has been demagnetized by a simple reversal of the field to a value apparently reducing B to zero, results in reversing the outer layer only, making the total flux zero *algebraically* as the sum of two *equal* and *opposite* fluxes in concentric layers.

Divided Ring Method. The divided ring method has the material in the form of a ring which has been cut in two and the opposite surfaces polished. The surfaces should be exactly in the same plane to insure a close fit when the ring is put together again. The pull necessary to separate the ring is twice as much as for one surface; so the total pull, after allowing for the weight of the upper part, should be divided by two before applying the previous formula. The ring is magnetized by a ring solenoid surrounding it. The solenoid is in two parts which separate with the parts of the ring.

Ballistic Method. If the material to be tested is in the form of a very long rod, say, 50 diameters in length, or better in the form of a ring, Fig. 65, with little difference between the outer and the inner radius, surrounded by a solenoid of n turns per cm. length through which the current I amp. passes, the field is $\mathbf{H} = 1.2566\ n\ I$. A secondary coil of a small number of turns (of which all or a part only may be used) is wound about the ring and is connected to a ballistic galvanometer whose throw for a known quantity passing is known. The ratio K of quantity to throw may be found by charging a condenser of known capacity C microfarads by a standard cell of e. m. f. E volts, and discharging it through a ballistic galvanometer, producing a deflection d. The constant K is then $K = \frac{E\ C}{d \times 10^6}$. Or the galvanometer constant may be determined by winding a solenoid of n_1 turns per cm. on a core of wood or other material of the same permeability as air, for which μ equals 1. H then equals B, and the flux passing through the core whose area is A_1, the current being I_1, is

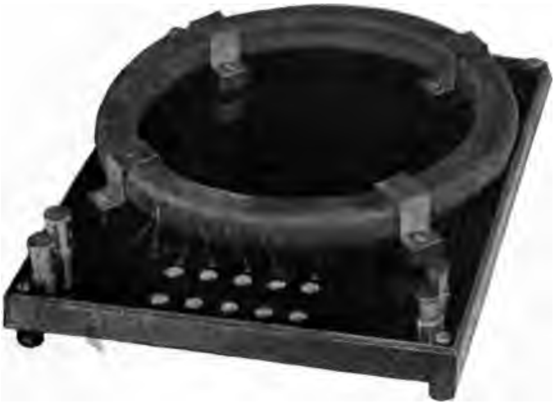

Fig. 65. Iron in Ring Form for Ballistic Measurement.

$$\Phi_1 = 1.2566\ n_1\ A_1\ I_1$$

The total quantity of electricity Q_1 passing through a ballistic galvanometer in series with a secondary circuit of m_1 turns and in a circuit of total resistance R_1, on making or breaking the primary circuit I_1, is

$$Q_1 = \frac{m_1\ \Phi_1}{R_1} = \frac{1.2566\ n_1\ m_1\ A_1\ I_1}{R_1}$$

If the deflection is d_1, the quantity per unit deflection is

$$\frac{Q}{d_1} = \frac{1.2566\ n_1\ m_1\ A_1\ I_1}{R\ d_1} = K$$

K is called the *constant* of the ballistic galvanometer.

If the primary circuit, the solenoid, has n turns per centimeter length, and the secondary circuit a total number m of turns, and the area of the section of the ring is A, and the resistance of the secondary circuit is R, and the deflection is D on reversing the primary circuit of I amperes, we have the flux

$$\Phi = \frac{R\,D\,K}{2\,m}$$

and

$$\mathrm{B} = \frac{\Phi}{A} = \frac{R\,D\,K}{2\,A\,m}$$

The current must be reversed suddenly, some form of commutator being used, otherwise the galvanometer may not feel the full effect. We had previously

$$\mathrm{H} = 1.2566\,n\,I$$

The value of the permeability for any value of H is

$$\mu = \frac{\mathrm{B}}{\mathrm{H}}$$

In using the method the current in the primary is *reversed* because otherwise the residual magnetism will produce a disturbance. If in the experiment the current starts at small values, the disturbing effects of previous magnetization is a minimum. We then insert a high resistance in the circuit at first and note the deflection of the galvanometer on reversing the current. Then increasing the current, the process is repeated. Several reversals should be made at each value of the current until enough observations have been made to ensure an accurate result. The deflection on the first reversal is apt to be different from those following. The B, H curve obtained by this method starts from the origin.

Hysteresis Curves. If instead of reversing the current it is simply changed by a sudden change in the resistance, the deflection will measure the change in B. Starting with no current in the primary and the ring unmagnetized, the circuit is suddenly closed with auxiliary resistance in circuit and the throw of the galvanometer noted as well as the current in the primary. The values of B and H are determined. Next the current is suddenly increased by cutting out part of the resistance and the deflection noted and the ammeter read. The deflection of the galvanometer measures the *increase* of B. The value of B is found by adding the *increase* to the previous value. The current is again increased and the deflection of the galvanometer will measure the increase of B. This is added to the previous total, and so on until the value of *B* is found for the highest value of H which is to be used. The current, and consequently H, is now reduced by steps, and the deflections of the galvanometer in the opposite direction will measure the decrease of B. To obtain the value of B, these decreases in B are subtracted from the previous value. When the zero value of the current, and consequently of H, is reached, the value of B will still be a considerable amount. The current is now reversed and built up by steps and the deflections will measure the continued decrease of B to zero and its reversal and building up in the reverse direction. This is continued until the maximum reverse value H is reached, equal, we will say, to its previous positive maximum. The current is reduced by steps to zero, reversed, and built up again in the first direction. The deflections measuring changes in B are noted and the total computed by the algebraic sum of all that precede. The sum of all the deflections corresponding to all the steps from the positive maximum to the negative maximum, should equal the sum in the reverse direction. The B, H curve will then make a closed curve for each cycle after the first quarter cycle. The curve for the first quarter cycle starts from the origin and never returns there again. As explained before, the magnetization lags behind the field H, producing the magnetization.

If the curve of B and H, as obtained by this method, is plotted, it is called a *hysteresis curve*, Fig. 61. The curve plotted to scale has an area to 4π times the energy in ergs expended per cubic centimeter per cycle. Dividing the area by $4 \times \pi \times 10^7$ gives the energy in joules per cycle. If the cycle were run through n times per second, the power in watts would be equal to $\frac{\text{area of curve} \times n}{40{,}000{,}000\ \pi}$ watts expended in each cubic centimeter. This is what happens when an alternating current is used.

There are two sources of error which may cause trouble in this experiment and which have not been mentioned above. The first is the effect of the current in the primary producing eddy currents in the material of the ring, just as currents are produced in the secondary circuit. In fact, the material of the ring is in itself a secondary circuit of a single turn, and currents in the material of the ring will be parallel to the current in the secondary outside. These eddy currents produce a magnetic field in the ring which opposes the rise of H and B in the ring. Therefore, H in the ring rises slower than the current in the primary coil; and unless the period of the galvanometer is high, some of the effect of the increase of B, which may be slow in increasing, will come too late to be measured by the galvanometer deflection. The other trouble is that if there is any vibration the magnetization may change by small steps one for each shock. This causes the magnetization to creep up or down, depending on whether the field has last been increased or decreased. To avoid the first error it is well to have the material in the form of thin sheets which give little chance for eddy currents, and to avoid the second the ring should rest on a pad of felt or other material which will absorb the vibrations.

Hysteresis Tester. A method of comparing the hysteresis loss of different samples of iron is to compare their effect in dragging the magnetic field when samples are rotated in a constant magnetic field Suppose the sample takes the form of a disk between a pair of field magnet poles. If the disk is at rest the field will produce a flux density B in the disk parallel to H. If the disk is now rotated the residual magnetization will cause the flux to rotate with the disk until the tangential component arrests further rotation of the flux. H and B then make a small angle with one another for very soft iron in the disk, and a proportionately larger angle for harder iron which shows more hysteresis. The turning moment required to rotate the disk will be proportional to the energy expended per cycle. If the poles are free to turn, they will follow the disk. If the poles are kept from rotating by some counter moment due to springs or gravitational action, the displacement of the poles in the direction of rotation of the disk will measure the relative hysteresis loss. If with one sample disk the displacement is twice that produced by another sample, the hysteresis loss is about double. If the hysteresis loss is known for one disk that of the other may be computed.

As the relative twist of B with respect to H is the same for all moderate speeds of rotation, it is not necessary to be careful about the exact speed of rotation. Also it is not necessary that the sample be in the form of a disk. The same relation for samples in the form of bundles of equal size strips will be found to hold true. The absolute angles of twist will be different, but they will have a ratio of equal value.

Professor Ewing has devised a hysteresis tester in which the sample, in the form of a bundle of strips, is rotated by means of a crank and gear train between the poles of a permanent magnet which is mounted on knife edges at a point above its center of gravity. The magnet follows the rotating bundle until the gravitational force gives an equal torque in the opposite direction.

If the sample is in the form of a solid bar, eddy currents of considerable magnitude may be produced which will complicate the results and introduce more or less uncertainty. Moreover, the eddy currents will be greater at higher than at lower rates of rotation, thus introducing different corrections at different speeds. For these reasons the bundle must be well laminated to obtain reliable results.

EXAMINATION PAPER